

# Hyperbolic Partial Differential Equations and Related Topics

**Date:** 26–27, January 2018

**Venue:** 61125 Room, 11th Floor, Building No.6, Chuo University  
1-13-27 Kasuga Bunkyo-ku Tokyo 112-8551 Japan  
中央大学 6 号館 11 階 25 号室 (61125)

## Program

### 26 January (Friday)

- 11:00 – 11:50 Taeko Yamazaki (Tokyo University of Science)  
Global existence and asymptotic profile of solutions to  
semilinear wave equations with structural damping
- 12:00 – 12:50 Mitsuhiro Nakao (Kyushu University)  
Global existence to the initial-boundary value problem  
for a system of semilinear wave equations

### Lunch 12:50 – 14:30

- 14:30 – 15:20 Kiyoshi Mochizuki (Tokyo Metropolitan University and  
Chuo University)  
An introduction to the scattering theory -a stationary  
approach-
- 15:40 – 16:30 Tsukasa Iwabuchi (Tohoku University)  
Besov spaces on open sets with the Dirichlet boundary  
condition and an application to the fractional Laplacian
- 16:50 – 17:40 Mitsuru Sugimoto (Nagoya University)  
A local-to-global boundedness argument and Fourier in-  
tegral operators

**27 January (Saturday)**

- 11:00 – 11:50    Piero D’Ancona (University of Roma)  
                          On the nonlinear Dirac equation with an electromagnetic potential
- 12:00 – 12:50    Hideo Kubo (Hokkaido University)  
                          Global existence for nonlinear wave equations with spatially dependent damping term

**Lunch 12:50 – 14:30**

- 14:30 – 15:20    Michael Ruzhansky (Imperial College London)  
                          Very weak solutions to hyperbolic equations
- 15:40 – 16:30    Tohru Ozawa (Waseda University)  
                          Lifespan of blowup solutions to the nonlinear Schrödinger equations on torus
- 16:50 – 17:40    Vladimir Georgiev (University of Pisa and Waseda University)  
                          On wave equation outside trapping obstacles and polynomial local energy decay

**Celebration 18:30 – 21:00****Support:**

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**Organizers:**

Tsukasa Iwabuchi (Tohoku University)  
 Koichi Taniguchi (Chuo University)

## Global existence and asymptotic profile of solutions to semilinear wave equations with structural damping

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We consider the Cauchy problem for the semilinear wave equation with structural damping

$$\begin{cases} u_{tt} + (-\Delta)^\sigma u_t - \Delta u = f(u), & t \geq 0, x \in \mathbb{R}^n, \\ u(0, x) = u_0(x), & x \in \mathbb{R}^n, \\ u_t(0, x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \quad (1)$$

Here we assume that  $f(u) \in C^k(\mathbb{R})$  with an integer  $k \in [0, p]$  and there is a positive constant  $C$  such that

$$\begin{cases} |\frac{d^j}{du^j} f(u)| \leq C|u|^{p-j} & (0 \leq j < k), \\ |\frac{d^k}{du^k} (f(u) - f(v))| \leq C|u - v|(|u| + |v|)^{p-k-1}. \end{cases} \quad (2)$$

D'Abbicco-Reissig [3] showed global existence and decay estimates of the solution of structural damped wave equations (1) with small data for space dimension  $n = 2, 3, 4$  and  $p \in [2, n/(n-2)]$  such that

$$p > p_\sigma := 1 + \frac{2}{n-2\sigma}. \quad (3)$$

Then D'Abbicco-Ebert [1] showed the diffusion phenomena for linear wave equations with structural damping (1) with  $f \equiv 0$ . They applied in [2] the results above to semilinear equations and showed the unique existence of solutions to (1) with small data in some Sobolev spaces for  $p > p_\sigma$  and space dimension  $n \leq \bar{n}(\sigma)$ , where  $\bar{n}(\sigma) \uparrow \infty$  as  $\sigma \uparrow 1/2$ .

The purpose of this talk is to show the unique existence of global solutions to (1) with  $p > p_\sigma$  and initial data in some classes for all space dimension  $n \geq 2$  without restriction from above. Furthermore, we give asymptotic profile of the solutions as  $t \rightarrow \infty$ .

## BIBLIOGRAPHY

- [1] M. D’Abbicco and M. R. Ebert, *Diffusion phenomena for the wave equation with structural damping in the  $L^p - L^q$  framework*, J. Differential Equations **256** (2014), 2307–2336.
- [2] M. D’Abbicco and M. R. Ebert, *An application of  $L^p - L^q$  decay estimates to the semi-linear wave equation with parabolic-like structural damping*, Nonlinear analysis **99** (2014) 16–34.
- [3] M. D’Abbicco and M. Reissig, *Semilinear structural damped waves*, Math. Methods Appl. Sci. **37** (2014), 1570–1592.

## Global existence to the initial-boundary value problem for a system of semilinear wave equations

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We consider the initial-boundary value problem for a system of nonlinear wave equations of the form:

$$u_{tt} - \Delta u + \rho(x, u_t) = f(u, v) \text{ in } \Omega \times (0, \infty), \quad (1.1)$$

$$v_{tt} - \Delta v = g(u, v) \text{ in } \Omega \times (0, \infty), \quad (1.2)$$

with the initial-boundary conditions

$$\begin{aligned} u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), v(x, 0) = v_0(x), v_t(x, 0) = v_1(x) \\ \text{and } u(x, t)|_{\partial\Omega} = v(x, t)|_{\partial\Omega} = 0, \end{aligned} \quad (1.3)$$

where  $\Omega$  is a bounded domain in  $R^N$ ,  $\rho(x, w)$  is a function like  $\rho(x, w) \approx a(x)|w|^r w$  near  $w = 0$ . The cases : (1)  $a(x) = \text{const.} > 0$  and (2)  $a(x) > \epsilon_0 > 0$  near a certain part of the boundary are considered. We discuss on the global existence of energy finite solution pair  $(u, v)$  and the so-called  $H_2$  solution pair  $(u, v)$  under appropriate smallness conditions on the initial data.

The source terms  $f(u, v)$  and  $g(u, v)$  are of the form  $f(u, v) \approx |u|^{\alpha-1}u|v|^{\max\{\beta-1, 0\}}v$  and  $g(u, v) \approx |u|^{p-1}u|v|^{\max\{q-1, 0\}}v$ , respectively. For our purpose we utilize the decay property of the solution of the first equation.

(The result is published in Nonlinear Analysis TMA, 146(2016), 233-257.)

## An introduction to the scattering theory - a stationary approach-

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This talk does not include new results. For short-range problems we summarize the existence, completeness and invariance properties of the Møller wave operators. A stationary approach is employed to give a unified proof to these properties.

The scattering theory compares two operators. Let  $L_0$  and  $L$  be self-adjoint operators respectively in the Hilbert spaces  $\mathcal{H}_0$  and  $\mathcal{H}$ . For some identification operator  $J \in \mathcal{B}(\mathcal{H}_0, \mathcal{H})$  if the strong limits

$$W_{\pm} = s - \lim_{t \rightarrow \pm\infty} e^{itL} J e^{-itL_0}$$

exist, we call them Møller wave operators. Moreover, if  $\mathcal{R}(W_+) = \mathcal{R}(W_-)$ , there exists the scattering operator

$$S = W_+^* W_-$$

which gives a unitary operator in  $\mathcal{H}_0$ . The invariance property asserts the following:

$$W_{\pm} = s - \lim_{t \rightarrow \pm\infty} e^{it\varphi(L)} J e^{-it\varphi(L_0)}$$

for each monotone increasing function  $\varphi(\lambda)$  of  $\lambda \in \mathbf{R}$ .

This note is summarized based on the recent book [1]. As for the growth properties of the generalized eigenfunctions (Lemma 3.3), a simpler form and proof have been introduced in [2].

[1] K. Mochizuki, *Spectral and Scattering Theory for Second Order Elliptic Differential Operators*, Monographs and Research Notes in Mathematics, CRC Press, Boca Raton London New York 2017 (Chapter 4,5,6).

[2] K. Mochizuki *Growth estimates of generalized eigenfunctions and principle of limiting absorption*, RIMS, Kokyuroku 2045 (2017), 85-99.

# Besov spaces on open sets with the Dirichlet boundary condition and an application to the fractional Laplacian

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Let  $\Omega$  be an arbitrary open set of  $\mathbb{R}^n$  with  $n \geq 1$ . We consider the Dirichlet Laplacian  $A$  on  $L^2(\Omega)$ , namely,

$$\begin{cases} D(A) := \{f \in H_0^1(\Omega) \mid \Delta f \in L^2(\Omega)\}, \\ Af := -\Delta f, \quad f \in D(A), \end{cases}$$

where  $\Delta := \partial_{x_1}^2 + \partial_{x_2}^2 + \cdots + \partial_{x_n}^2$ . Since  $A$  is self-adjoint, we can apply the spectral theorem to be able to define  $A^{s/2}$  for  $s \in \mathbb{R}$ . One can define the Sobolev spaces  $H^s(A)$  generated by the Dirichlet Laplacian as follows:

$$H^s(A) := \{f \in L^2(\Omega) \mid A^{s/2}f \in L^2(\Omega)\} \quad \text{for } s \geq 0.$$

In this talk, we consider such spaces of Besov type and introduce the Besov spaces of the inhomogeneous type and the homogeneous type on  $\Omega$  based on the spectral theory. An application of that theory to the fractional Laplacian will be also shown.

We introduce the partition of unity, the definition of Besov spaces with spaces of distributions and state theorems. Let  $\psi, \phi_j \in C_0^\infty(\mathbb{R})$  ( $j \in \mathbb{Z}$ ) be non-negative functions on  $\mathbb{R}$  such that

$$\begin{cases} \text{supp } \phi_0 \subset [2^{-1}, 2], \quad \phi_j(\lambda) = \phi_0(2^{-j}\lambda) \text{ for } \lambda \in \mathbb{R}, \\ \psi(\lambda) + \sum_{j \in \mathbb{N}} \phi_j(\lambda) = 1 \text{ for } \lambda \geq 0, \quad \sum_{j \in \mathbb{Z}} \phi_j(\lambda) = 1 \text{ for } \lambda > 0. \end{cases} \quad (1)$$

**Definition.** (i)  $\mathcal{X}(A)$  is defined by

$$\mathcal{X}(A) := \{f \in L^1(\Omega) \cap \mathcal{D}(A) \mid A^M f \in L^1(\Omega) \cap \mathcal{D}(A) \text{ for all } M \in \mathbb{N}\}$$

equipped with the family of semi-norms  $\{p_{A,M}(\cdot)\}_{M=1}^\infty$  given by

$$p_{A,M}(f) := \|f\|_{L^1(\Omega)} + \sup_{j \in \mathbb{N}} 2^{Mj} \|\phi_j(\sqrt{A})f\|_{L^1(\Omega)}.$$

(ii)  $\mathcal{Z}(A)$  is defined by

$$\mathcal{Z}(A) := \left\{ f \in \mathcal{X}(A) \mid \sup_{j \leq 0} 2^{M|j|} \|\phi_j(\sqrt{A})f\|_{L^1(\Omega)} < \infty \text{ for all } M \in \mathbb{N} \right\}$$

equipped with the family of semi-norms  $\{q_{A,M}(\cdot)\}_{M=1}^\infty$  given by

$$q_{A,M}(f) := \|f\|_{L^1(\Omega)} + \sup_{j \in \mathbb{Z}} 2^{M|j|} \|\phi_j(\sqrt{A})f\|_{L^1(\Omega)}.$$

(iii)  $\mathcal{X}'(A), \mathcal{Z}'(A)$  denote the topological duals of  $\mathcal{X}(A), \mathcal{Z}(A)$ .

**Definition.** Let  $s \in \mathbb{R}$  and  $1 \leq p, q \leq \infty$ .

(i)  $B_{p,q}^s(A)$  is defined by

$$B_{p,q}^s(A) := \{f \in \mathcal{X}'(A) \mid \|f\|_{B_{p,q}^s(A)} < \infty\},$$

where

$$\|f\|_{B_{p,q}^s(A)} := \|\psi(\sqrt{A})f\|_{L^p(\Omega)} + \left\| \{2^{sj} \|\phi_j(\sqrt{A})f\|_{L^p(\Omega)}\}_{j \in \mathbb{N}} \right\|_{\ell^q(\mathbb{N})}.$$

(ii)  $\dot{B}_{p,q}^s(A)$  is defined by letting

$$\dot{B}_{p,q}^s(A) := \{f \in \mathcal{Z}'(A) \mid \|f\|_{\dot{B}_{p,q}^s(A)} < \infty\},$$

where

$$\|f\|_{\dot{B}_{p,q}^s(A)} := \left\| \{2^{sj} \|\phi_j(\sqrt{A})f\|_{L^p(\Omega)}\}_{j \in \mathbb{Z}} \right\|_{\ell^q(\mathbb{Z})}.$$

**Theorem ([1]).** Let  $s \in \mathbb{R}$  and  $1 \leq p, q \leq \infty$ .

(i)  $B_{p,q}^s(A)$  and  $\dot{B}_{p,q}^s(A)$  are independent of the choice of the partition of unity satisfying (1). The following continuous embedding hold:

$$\mathcal{X}(A) \hookrightarrow B_{p,q}^s(A) \hookrightarrow \mathcal{X}'(A), \quad \mathcal{Z}(A) \hookrightarrow \dot{B}_{p,q}^s(A) \hookrightarrow \mathcal{Z}'(A),$$

(ii)  $B_{p,q}^s(A)$  and  $\dot{B}_{p,q}^s(A)$  are Banach spaces.

(iii) If  $1 \leq p, q < \infty$ , then dual spaces of  $B_{p,q}^s(A), \dot{B}_{p,q}^s(A)$  are  $B_{p',q'}^{-s}(A), \dot{B}_{p',q'}^{-s}(A)$ , respectively, where  $p', q'$  are such that  $1/p + 1/p' = 1/q + 1/q' = 1$ .

(iv) If  $1 \leq r \leq p$ , then the following embedding hold:

$$B_{r,q}^{s+n(\frac{1}{r}-\frac{1}{p})}(A) \hookrightarrow B_{p,q}^s(A), \quad \dot{B}_{r,q}^{s+n(\frac{1}{r}-\frac{1}{p})}(A) \hookrightarrow \dot{B}_{p,q}^s(A)$$

[1] T. Iwabuchi, T. Matsuyama, K. Taniguchi, *Besov spaces on open sets*, preprint, arXiv:1603.01334v1.



## A local-to-global boundedness argument and Fourier integral operators

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We discuss when the local boundedness of an integral operator induces the global one. Let  $\mathcal{K}$  be an integral operator of the form

$$\mathcal{K}u(x) = \int_{\mathbb{R}^n} K(x, y, x - y)u(y) dy \quad (x \in \mathbb{R}^n) \quad (1)$$

with a measurable function  $K(x, y, z)$  on  $\mathbb{R}_x^n \times \mathbb{R}_y^n \times \mathbb{R}_z^n$ . The formal adjoint  $\mathcal{K}^*$  of  $\mathcal{K}$  is given by

$$\mathcal{K}^*u(x) = \int_{\mathbb{R}^n} K^*(x, y, x - y)u(y) dy, \quad K^*(x, y, z) = \overline{K(y, x, -z)}. \quad (2)$$

We introduce a notion of the local boundedness. By  $\chi_B$  we denote the multiplication by the smooth characteristic function of the set  $B \subset \mathbb{R}^n$ .  $H^1(\mathbb{R}^n)$  denotes the Hardy space introduced by Fefferman-Stein.

**Definition.** We say that the operator  $\mathcal{K}$  is  $H_{comp}^1(\mathbb{R}^n)$ - $L_{loc}^1(\mathbb{R}^n)$ -bounded if the localised operator  $\chi_B \mathcal{K} \chi_B$  is  $H^1(\mathbb{R}^n)$ - $L^1(\mathbb{R}^n)$ -bounded for any compact set  $B \subset \mathbb{R}^n$ . Furthermore, if the operator norm of  $\chi_{B_h} \mathcal{K} \chi_{B_h}$  is bounded in  $h \in \mathbb{R}^n$  for the translated set  $B_h = \{x + h : x \in B\}$  of any compact set  $B \subset \mathbb{R}^n$ , i.e. if

$$\sup_{h \in \mathbb{R}^n} \|\chi_{B_h} \mathcal{K} \chi_{B_h}\|_{H^1(\mathbb{R}^n) \rightarrow L^1(\mathbb{R}^n)} < \infty,$$

we say that the operator  $\mathcal{K}$  is uniformly  $H_{comp}^1(\mathbb{R}^n)$ - $L_{loc}^1(\mathbb{R}^n)$ -bounded.

If we introduce the translation operator  $\tau_h : f(x) \mapsto f(x - h)$  and its inverse (formal adjoint)  $\tau_h^* = \tau_{-h}$ , we have the equality  $\chi_{B_h} = \tau_h \chi_B \tau_h^*$ . Since  $L^1$  and  $H^1$  norms are translation invariant,  $\mathcal{K}$  is uniformly  $H_{comp}^1$ - $L_{loc}^1$ -bounded if and only if  $\chi_B (\tau_h^* \mathcal{K} \tau_h) \chi_B$  is  $H^1$ - $L^1$ -bounded for any compact set  $B \subset \mathbb{R}^n$  and the operator norms are bounded in  $h \in \mathbb{R}^n$ . We remark that the operator  $\tau_h^* \mathcal{K} \tau_h$  has the expression

$$\tau_h^* \mathcal{K} \tau_h u(x) = \int_{\mathbb{R}^n} K_h(x, y, x - y)u(y) dy, \quad K_h(x, y, z) = K(x + h, y + h, z) \quad (3)$$

We have the following main result:

**Main Theorem.** *Suppose that operator  $\mathcal{K}$  defined by (1) is  $L^2(\mathbb{R}^n)$ -bounded and uniformly  $H_{comp}^1(\mathbb{R}^n)$ - $L_{loc}^1(\mathbb{R}^n)$ -bounded. Assume that there exists a measurable function  $H(x, y, z)$  which satisfies the following condition:*

(A1) *There exist constants  $d > 0$  and  $k > n$  such that*

$$\sup_{H(x,y,z) \geq d} \left| H(x, y, z)^k K(x, y, z) \right| < \infty.$$

*Furthermore, we set*

$$\tilde{H}(z) := \inf_{x,y \in \mathbb{R}^n} H(x, y, z).$$

*and assume also the following two conditions:*

(A2) *There exist constants  $A > 0$  and  $A_0 > 0$  such that*

$$\tilde{H}(z) \geq A_0 |z| \text{ whenever } |z| \geq A.$$

(A3) *There exist constants  $b > 0$  and  $b_0 > 0$  such that*

$$\tilde{H}(z) \leq b_0 \tilde{H}(z - z') \text{ whenever } \tilde{H}(z) \geq b |z'|.$$

*Then  $\mathcal{K}$  is  $H^1(\mathbb{R}^n)$ - $L^1(\mathbb{R}^n)$ -bounded. If in addition operator  $\mathcal{K}^*$  defined by (2) is uniformly  $H_{comp}^1(\mathbb{R}^n)$ - $L_{loc}^1(\mathbb{R}^n)$ -bounded, then  $\mathcal{K}^*$  is also  $H^1(\mathbb{R}^n)$ - $L^1(\mathbb{R}^n)$ -bounded.*

Main theorem means that the global  $L^2$ -boundedness and some additional assumptions induce the global  $H^1$ - $L^1$ -boundedness from the local one. Then, if we want, we can have the global  $L^p$ -boundedness for  $1 < p < \infty$  by the interpolation and the duality argument.

As an application, we will also discuss the global boundedness of Fourier integral operators of a certain class.

This is joint work with Michael Ruzhansky (Imperial College London).

## On the nonlinear Dirac equation with an electromagnetic potential

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In a joint work with M.Okamoto (Shinshu University, Nagano) we prove smoothing and Strichartz estimates for a Dirac equation perturbed by a large potential of critical decay and regularity. In the endpoint case, we prove suitable replacements of these estimates for data of additional angular regularity. As an application we deduce global well posedness and scattering for small data in the energy space with radial symmetry, or with additional angular regularity. Moreover, for a restricted class of potentials, we can extend our results to more general large data under the sole assumption of smallness of the Lochak-Majorana chiral invariants.

- [1] P.D'Ancona, M.Okamoto, Blowup and ill-posedness results for the Dirac equation without gauge invariance, *Evolution Equations and Control Theory* **5** (2016) 225–234
- [2] P.D'Ancona, M.Okamoto, On the cubic Dirac equation with potential and the Lochak–Majorana condition, *J. Math. Anal. Appl.* **456** (2017) 1203–1237 [<https://arxiv.org/abs/1706.06479>]
- [3] P.D'Ancona, Kato smoothing and Strichartz estimates for wave equations with magnetic potentials, *Comm. Math. Phys.* **335** (2015) 1–16 [<https://arxiv.org/abs/1403.2537>]

## Global existence for nonlinear wave equations with spatially dependent damping term

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In this talk I'd like to present a result on the global solvability for nonlinear wave equations with spatially dependent damping term:

$$(\partial_t^2 + 2w_*(x)\partial_t - \Delta + V_*(x))v = (\partial_t v)^2 \quad \text{in } (0, T) \times \mathbb{R}^3, \quad (1)$$

$$v(0, x) = \varepsilon f_0(x), \quad (\partial_t v)(0, x) = \varepsilon f_1(x) \quad \text{for } x \in \mathbb{R}^3, \quad (2)$$

where  $f_0$ ,  $f_1$ ,  $w_*$ , and  $V_*$  are assumed to be radially symmetric functions in  $\mathbb{R}^3$ . There exists huge literatures on the damped wave equations, but the pointwise decay property for the above type of equation have not yet examined, up to my knowledge. When the nonlinear term is semilinear, that is,  $|u|^p$ , the corresponding initial value problem is considered by Georgiev and Kubo[1], recently. They showed that the critical exponent for the problem is  $p = (3 + \sqrt{17})/4$ , which corresponds to the one for the pure wave equation in five space dimensions. This means that the damping term has an effect of the shift of the critical exponent. Therefore, it seems natural to ask whether the same effect can be observed for the case of  $(\partial_t u)^2$  or not. Note that without the damping term, the solution to the corresponding problem blows up in a finite time, and some algebraic structure is necessary to get the global solvability for the small initial data. We assume that

$$V_*(x) = -x \cdot \nabla w_*(x) + w_*(x)^2 \quad \text{for } x \in \mathbb{R}^3 \quad (3)$$

with a positive decreasing function  $w_*(x)$  satisfying

$$w_*(x) = \frac{1}{|x|} \quad \text{for } |x| \geq 1. \quad (4)$$

Then we can show that the solutions to the corresponding linear equation has the explicit representation formula, and hence we can obtain pointwise decay estimates. Based on the estimates, we can prove that there exists a classical solution to the initial value problem (1)-(2), provided  $\varepsilon$  is sufficiently small, and  $f_0$  and  $f_1$  decay suitably fast.

[1] V. Georgiev, H. Kubo, in preparation.

## Very weak solutions to hyperbolic equations

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In this talk we will discuss the notion of ‘very weak solutions’ that was developed in [1] to deal with equations with irregular (distributional) coefficients. The existence of very weak solutions, their uniqueness, and consistency with more regular (classical, weak, distributional, ultradistributional) solutions if they exist, was shown in [1,2,3,4] for different types of Cauchy problems. We will also discuss the numerical experiments and discoveries, following [4].

- [1] Garetto C., Ruzhansky M., *Hyperbolic second order equations with non-regular time dependent coefficients*, Arch. Ration. Mech. Anal., 217 (2015), 113-154.
- [2] Ruzhansky M., Tokmagambetov N., *Wave equation for operators with discrete spectrum and irregular propagation speed*, Arch. Ration. Mech. Anal., 226 (2017), 1161-1207.
- [3] Ruzhansky M., Tokmagambetov N., *Very weak solutions of wave equation for Landau Hamiltonian with irregular electromagnetic field*, Lett. Math. Phys., 107 (2017), 591-618.
- [4] Munoz J. C., Ruzhansky M., Tokmagambetov N., *Wave propagation with irregular dissipation and applications to acoustic problems and shallow waters*, arxiv

## Lifespan of blowup solutions to the nonlinear Schrödinger equations on torus

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An explicit lifespan estimate is presented for nonlinear Schrödinger equations on one-dimensional torus. This talk is based on my recent joint-work with Kazumasa Fujiwara.

- [1] (with K. Fujiwara) *Lifespan of strong solutions to the periodic nonlinear Schrödinger equation without gauge invariance*, J. Evol. Equ., **17** (2017), 1023–1030.
- [2] (with K. Fujiwara) *Lifespan of strong solutions to the periodic derivative nonlinear Schrödinger equation*, preprint (2017).
- [3] T. Oh, *A blowup result for the periodic NLS without gauge invariance*, C. R. Math. Acad. Sci. Paris, **350** (2012), 389–392.
- [4] (with Y. Yamazaki) *Life-span of smooth solutions to the complex Ginzburg-Landau type equation on a torus*, Nonlinearity, **16** (2003), 2029–2034.

## On wave equation outside trapping obstacles and polynomial local energy decay

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Our goal is to establish the local energy decay estimates for 3-dimensional wave equation to the initial-boundary value problem on exterior domains. The geometrical assumptions on domains are rather general, for example non-trapping condition is not imposed. However, some geometrical assumptions on the behavior of the Hamiltonian flow near trapped set in the cosphere bundle is required. The work is a part of the collaboration work with Tokio Matsuyama and Koichi Taniguchi.