UK Network on Hyperbolic Equations and Related Topics, 2018-2019

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Departments

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Linear and nonlinear hyperbolic partial differential equations (PDEs) arise in basically all sciences (physics, chemistry, medicine, engineering, astronomy, etc.). In physics, they model several important phenomena, from propagation of waves in a medium (for instance propagation of seismic waves during an earthquake) to refraction in crystals and gas-dynamics. The purpose of this UK network on hyperbolic equations and related topics is to bring together the expertise on hyperbolic equations of three different mathematics department (Edinburgh, Imperial, Loughborough), to strengthen the existing research collaborations and to create new ones. Three 1-day workshops per year are planned focused on different approaches to hyperbolic equations and related topics (inverse problems, kinetic theory, imaging, microlocal analysis, general relativity, etc.).

Meeting 1

18 December 2018 (Tuesday), Queen Mary University London, Arts 1, Room 128

Planned program:

10:30-11:20: **Dmitri Vassiliev** (UCL)
Geometric wave propagator on Riemannian manifolds

11:20-12:10: **Claudia Garetto** (Loughborough)
Hyperbolic systems with non-diagonalisable principal part and variable multiplicities

12:10-13:00 **Shabnam Beheshti** (QMUL)
Well-Posedness of Viscous Cosmology, First Steps

13:00-13:30 TBA

LUNCH

14:30-15:20: **Valery Smyshlyaev** (UCL)
Two-scale homogenisation for a general class of high-contrast hyperbolic systems

15:20-16:10: **Emiliano Renzi** (Loughborough)
Hydro-acoustic waves in weakly compressible fluid

COFFEE

16:30-17:00 **Tom Baker** (Imperial)
Pseudo-differential Operators with Short Time Fourier Transforms and Gabor Frames

Abstracts

Dmitri Vassiliev (UCL)

Geometric wave propagator on Riemannian manifolds

The talk deals with the wave equation on a Riemannian manifold. The propagator is the operator which maps initial conditions to a solution of the wave equation. The goal is to construct the propagator explicitly, modulo an integral operator with infinitely smooth kernel. Here by "explicitly" we mean reducing the problem to integration of ordinary differential equations. It has been known since the 1950s that this goal can be achieved using microlocal techniques, however in its standard version this construction is local in space and in time and involves taking compositions of operators. It turns out that the propagator can be written as a single oscillatory integral, global in space and in time, and that this can be done in an invariant geometric fashion. The results presented in the talk are a development of earlier results of Ari Laptev, Yuri Safarov and the speaker. This is joint work with Matteo Capoferri and Michael Levitin.

Claudia Garetto (Loughborough)

Hyperbolic systems with non-diagonalisable principal part and variable multiplicities

In this talk I present some recent results obtained in collaboration with Michael Ruzhansky (Ghent/QMUL) and Christian Jäh (Göttingen) for a rather general class of hyperbolic systems with space-time dependent coefficients and multiple variable characteristics. The well-posedness of the corresponding Cauchy problem is proven in anisotropic Sobolev spaces under interesting natural conditions on the orders of lower order terms below the diagonal. We start by discussing systems with upper-triangular principal part and then we find suitable hypotheses which allow a reduction into upper-triangular form. Finally, we present a FIO representation formula of the solution when the principal part of the system is only space-dependent.

Shabnam Beheshti (QMUL)

Well-Posedness of Viscous Cosmology, First Steps

It is known that viscous effects lead to nontrivial dynamical behaviour in homogeneous cosmological models such as FLRW and Bianchi spacetimes. Recent progress in well-posedness of certain Einstein-Navier-Stokes systems motivates revisiting a model first proposed by Lichnerowicz in 1967. We investigate the role of dynamic velocity in a cosmological background, demonstrating that the additional degree of freedom afforded by the associated fluid index may play a geometric role in the associated evolution equations. First steps, numerical evidence and open questions will be discussed.

Valery Smyshlyaev (UCL)

Two-scale homogenisation for a general class of high-contrast hyperbolic systems

In this talk I review a recent work with Ilia Kamotski [1] on two-scale homogenisation of general PDE systems with periodic coefficients with a critically scaled high contrast, which reflects the effect of underlying "micro-resonances''. It appears that a strong two-scale resolvent convergence of associated high-contrast elliptic operators holds under a rather generic decomposition assumption. This implies in particular (two-scale) convergence of hyperbolic semigroups with applications to a wide class of hyperbolic initial value problems. The resulting two-scale limit hyperbolic problems display some interesting effects due to the micro-resonances.
In the end I briefly discuss situations where the micro-resonances display certain randomness. In simplest cases, the resulting two-scale limit behaviour appears to be rather explicit and the macroscopic equations display a form of wave trapping by the micro-resonances due to their randomness.

[1] I.V. Kamotski, V.P. Smyshlyaev, Two-scale homogenization for a general class of high contrast PDE systems with periodic coefficients, *Applicable Analysis* (2018).

Emiliano Renzi (Loughborough)
Hydro-acoustic waves in weakly compressible fluid

A novel analytical solution for hydro-acoustic waves in a weakly compressible fluid flow over a slowly varying bottom is presented. A multiple-scale perturbation technique and matched asymptotic analysis lead to a uniform analytical solution of the depth-averaged governing equations in three dimensions. We show that the slow depth variation has a leading-order effect on the evolution of the acoustic mode amplitude and direction. For the case of a plane beach, we find an exact analytical solution of the model equation in terms of integrals of Bessel functions. Our model offers an insight into the evolution of hydro-acoustic waves of interest for the design of tsunami early warning systems.

Tom Baker (Imperial)
Pseudo-differential Operators with Short Time Fourier Transforms and Gabor Frames

In this talk I will discuss alternate representations of the Hörmander class using Time- Space Frequency analysis, which we then can generalise. In particular, with the Short Time Fourier transform we consider symbols with unbounded growth in x and develop a calculus over these operators. Using this we can construct a form of parametrix for some non-elliptic pseudo-differential operators.
Using Gabor frames, we see how the class an operator belongs to is determined by the growth of its Gabor matrix. A system of difference operators is also introduced to describe these matrices.