UK Network on Hyperbolic Equations and Related Topics, 2019-2020

Supported by LMS (Scheme 3) and by the Edinburgh Mathematical Society

Departments

School of Mathematics, The University of Edinburgh, Maxwell Institute for Mathematical Sciences

Department of Mathematical Sciences, Loughborough University

School of Mathematical Sciences, Queen Mary University London

Organisers

Pieter Blue (Edinburgh)
Claudia Garetto (Loughborough)
Michael Ruzhansky (Queen Mary London)

Linear and nonlinear hyperbolic partial differential equations (PDEs) arise in basically all sciences (physics, chemistry, medicine, engineering, astronomy, etc.). In physics, they model several important phenomena, from propagation of waves in a medium (for instance propagation of seismic waves during an earthquake) to refraction in crystals and gas-dynamics. The purpose of this UK network on hyperbolic equations and related topics is to bring together the expertise on hyperbolic equations of three different mathematics department (Edinburgh, Imperial, Loughborough), to strengthen the existing research collaborations and to create new ones. Three 1-day workshops per year are planned focused on different approaches to hyperbolic equations and related topics (inverse problems, kinetic theory, imaging, microlocal analysis, general relativity, etc.).

Meeting 1

19 February 2020 (Wednesday), Loughborough University, Room JCR001

Planned program

10:00-10:30 ARRIVAL AND COFFEE

10:30-11:20 **Pieter Blue** (Edinburgh)
On the stability of higher dimensions

11:20-12:10 **Rita Teixeira da Costa** (Cambridge)
Mode stability for the Teukolsky equation on extremal Kerr black hole spacetimes

12:10-13:00 **Dan Ratliff** (Loughborough)
Entirely Out Of Character? Moving Frames in Dispersive Dynamics

13:00-14:00 LUNCH

14:00-14:50 **Federica Dragoni** (Cardiff)
Γ- convergence and homogenisation for a class of degenerate functionals

14:50-15:40 **Emiliano Renzi** (Loughborough)
A depth-integrated model for large-scale modelling of low-frequency hydroacoustic waves

15:40-16:00 COFFEE AND CLOSURE

Abstracts

Pieter Blue (Edinburgh)
On the stability of higher dimensions

There is a large class of Kaluza-Klein type spaces given by the Cartesian product of $1+n$ dimensional Minkowski space with a Ricci-flat Riemannian manifold, called the internal space. These are solutions of the Einstein equation. This talk will show that these spaces are stable as solutions of the Einstein equation when n is sufficiently large and, at least, when the internal space is a torus. This requires taking the intersection of methods for quasilinear wave and Klein-Gordon equations. This stability result is a related to a conjecture of Penrose concerning the validity of string theory.

Rita Teixeira da Costa (Cambridge)
Mode stability for the Teukolsky equation on extremal Kerr black hole spacetimes

We prove that there are no exponentially growing modes nor modes on the real axis for the Teukolsky equation on extremal Kerr black hole spacetimes. The nature of the Kerr black hole event horizon changes radically in the extremal limit. It is for this reason that, although mode stability was already known for subextremal spacetimes, its proof cannot be extended to the extremal case and a new idea is needed.

We also explain how mode stability could serve as a preliminary step towards understanding boundedness, scattering and decay properties of general solutions to the Teukolsky equation on extremal Kerr black holes.

Dan Ratliff (Loughborough)
Entirely Out Of Character? Moving Frames in Dispersive Dynamics

In the evolution of nonlinear waves, localised structures and defects can form and persist, even within stable waves. One way that their formation can be understood is by using the Whitham Modulation equations (WMEs), a dispersionless set of quasilinear PDEs. However, a persistent problem is how to regularise this system via the inclusion of dispersive effects to prevent the emergence of multivalued wave quantities. Surprisingly, it transpires that such features already lurk within the WMEs whenever they are hyperbolic – one merely waits long enough in a suitable moving frame. This takes the form of the Korteweg – de Vries (KdV) equation and is universal in the sense that its coefficients are tied to abstract properties of the original Lagrangian.
This leads to a more general question – can the properties of the characteristics be used to infer the resulting dynamics? This talk confirms this, and the connection between established concepts in hyperbolic systems (such as the Hamiltonian-Hopf bifurcation and linear degeneracy) and some well-known nonlinear dispersive equations, such as the Two-Way Boussinesq and modified KdV equations, are made.

Federica Dragoni (Cardiff)
Γ- convergence and homogenisation for a class of degenerate functionals

I will present a Γ-convergence for degenerate integral functionals related to homogenisation problems in the Heisenberg group. In our case, both the rescaling and the notion of invariance or periodicity are chosen in a way

motivated by the geometry of the Heisenberg group. Without using special geometric features, these functionals would be neither coercive nor periodic, so classic results do not apply. All the results apply to the more general case of Carnot groups. Joint with Nicolas Dirr, Paola Mannucci and Claudio Marchi.

Emiliano Renzi (Loughborough)

A depth-integrated model for large-scale modelling of low-frequency hydroacoustic waves

We present a depth-integrated model for the dynamics of propagation of underwater acoustic waves that are co-generated with surface gravity waves by forcing at the boundary of the ocean. This forcing can be physically associated with bottom movements (e.g. earthquakes and landslides) as well as surface pressure changes due to storms. We discuss the main features of the depth-integrated equation both analytically and numerically and show practical applications where the model can be used to predict the arrival of a tsunami.