Book "Quantization on Nilpotent Lie Groups"

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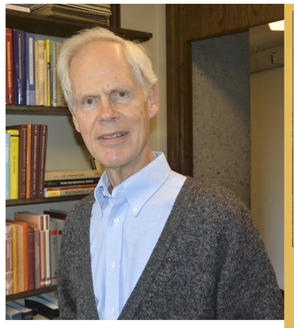
Description

This open access book presents a consistent development of the Kohn-Nirenberg type global quantization theory in the setting of graded nilpotent Lie groups in terms of their representations. It contains a detailed exposition of related background topics on homogeneous Lie groups, nilpotent Lie groups, and the analysis of Rockland operators on graded Lie groups together with their associated Sobolev spaces. For the specific example of the Heisenberg group the theory is illustrated in detail. In addition, the book features a brief account of the corresponding quantization theory in the setting of compact Lie groups.

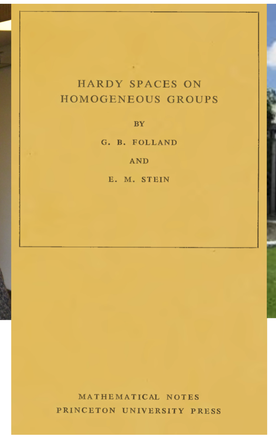
$$\mathrm{Op}(\sigma)f(x) = \int_{\widehat{\mathbb{G}}} \mathrm{Tr}\left(\pi(x)\,\sigma(x,\pi)\,\widehat{f}(\pi)\right) d\mu(\pi)$$

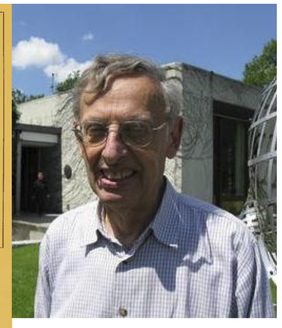
Content

Hardy Spaces on Homogeneous Groups



Gerald Budge Folland is an American mathematician and a professor of mathematics at the University of Washington. Picture from Wikipedia.





Elias Menachem Stein (January 13, 1931 – December 23, 2018) was an American mathematician who was a leading figure in the field of harmonic analysis. Picture from Wikipedia.

G. B. Folland and E. M. Stein: "That (homogeneous) groups form a natural habitat for extensions of many of the objects studied in Euclidean harmonic analysis can be understood from several points of view. First, it is a trite observation that many of the basic operators of harmonic analysis in $\mathbb{R}^n$ are of the convolution type; and in addition these have some natural invariance with respect to dilations. If we seek to generalize this situation we are naturally led to groups which have appropriate one-parameter groups of automorphisms acting like dilations. These are the homogeneous groups."

Rockland operator

Rockland condition: Let T be a left-invariant differential operator on a Lie group $\mathbb{G}$. Then T satisfies the *Rockland condition* when

- for each representation $\pi \in \hat{\mathbb{G}}$, except for the trivial representation, the operator $\pi(T)$ is injective on the space of smooth vectors $\mathcal{H}^\infty_\pi$, that is,

$$\forall v \in \mathcal{H}^\infty_\pi \;\; \pi(T)v = 0 \Rightarrow v = 0,$$

  where $\pi(T)$ is the infinitesimal representation of T as an element of the universal enveloping algebra.

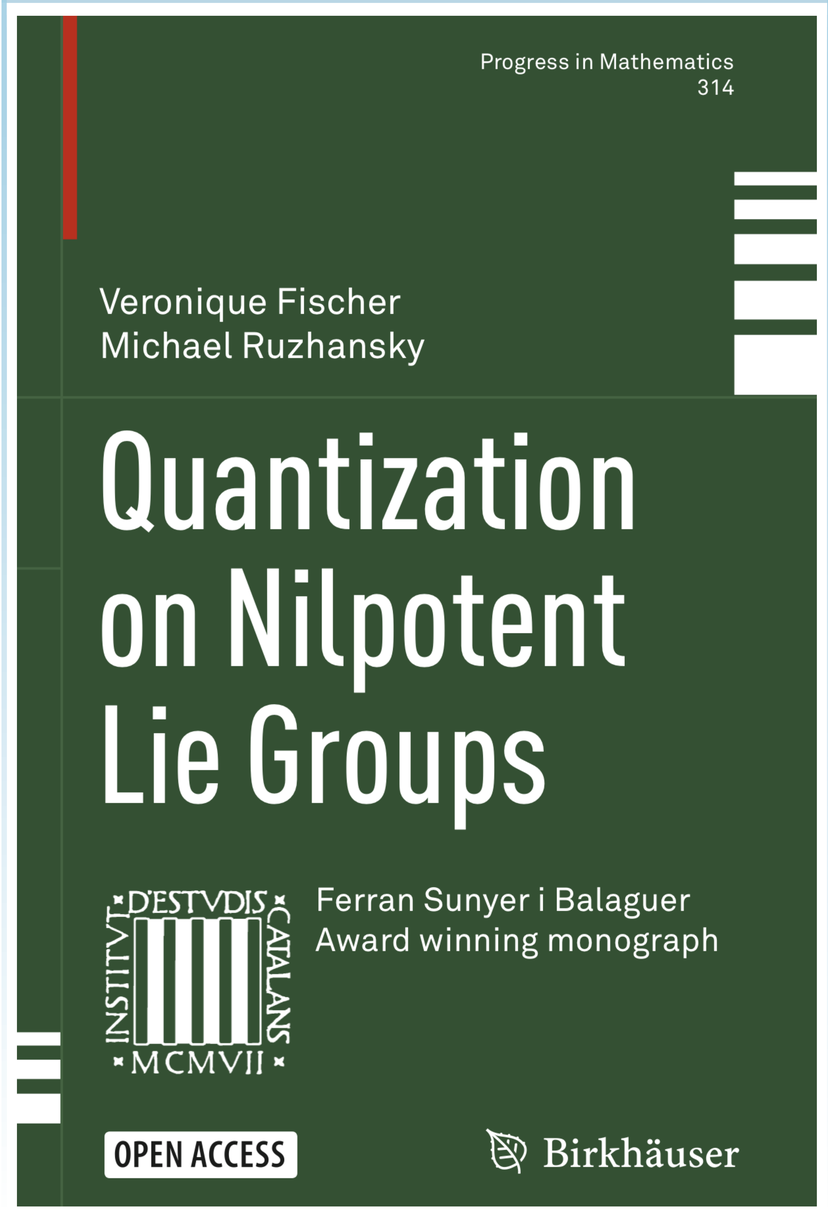
Rockland operator: Let $\mathbb{G}$ be a homogeneous Lie group. A *Rockland operator* $\mathcal{R}$ on $\mathbb{G}$ is a left-invariant differential operator which is homogeneous of positive degree and satisfies the Rockland condition.

Helffer-Nourrigat: A left invariant homogeneous differential operator is a Rockland operator if and only if it is hypoelliptic.

Ferran Sunyer i Balaguer Prize

This monograph is **the winner of the 2014 Ferran Sunyer i Balaguer Prize**, a prestigious award for books of expository nature presenting the latest developments in an active area of research in mathematics, where the authors made an important contribution.

Open Access Book



Information

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