Imperial College London



Book "Quantization on Nilpotent Lie Groups"

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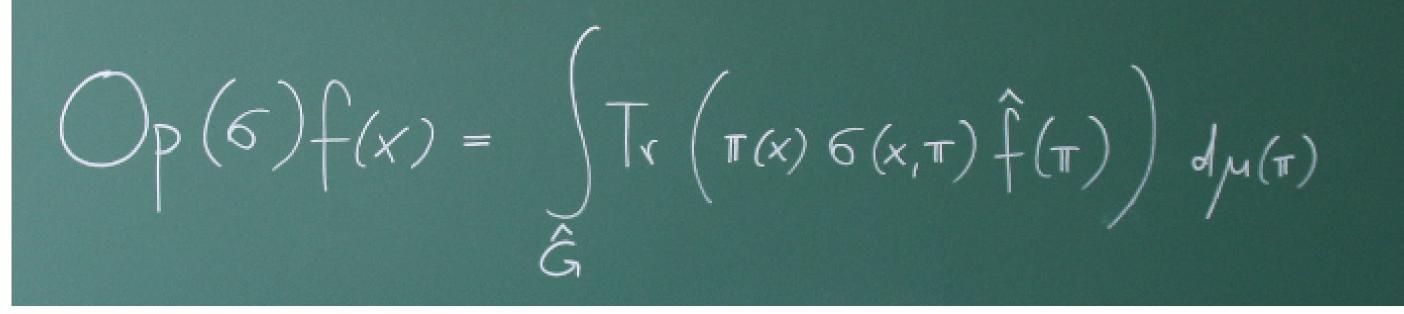
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Description

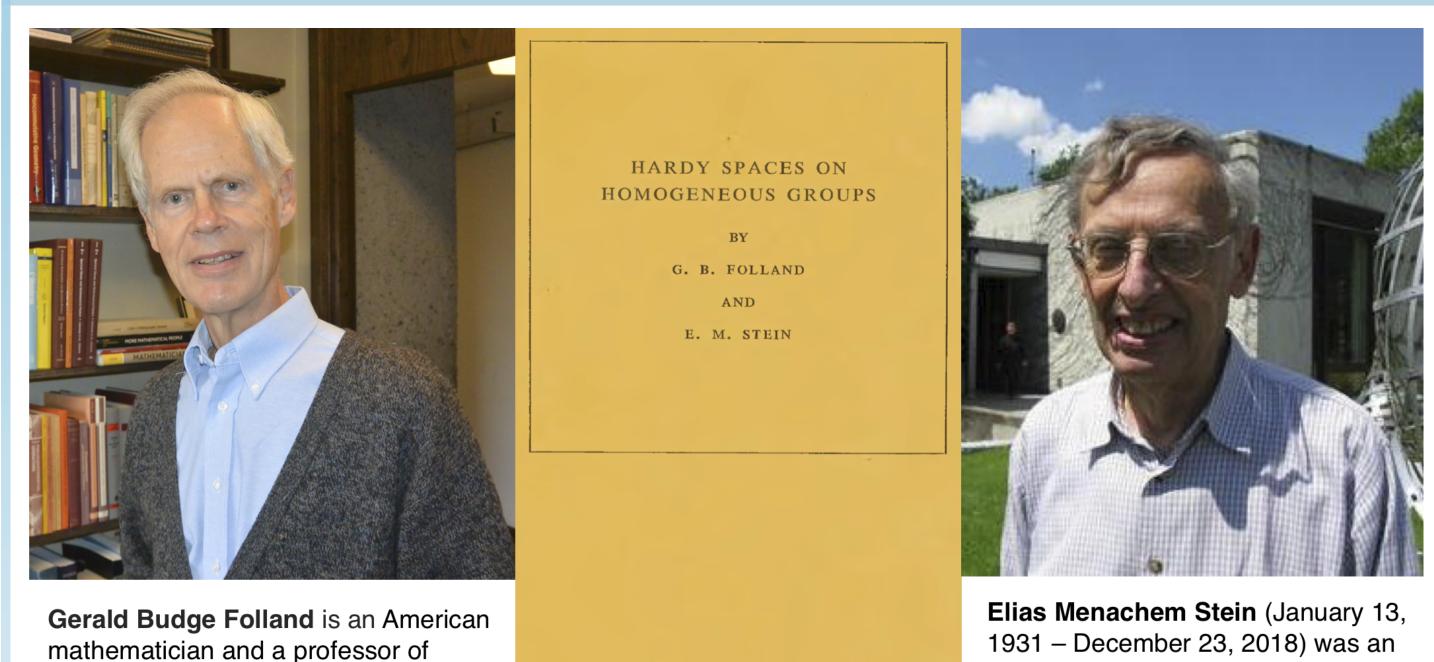
This open access book presents a consistent development of the Kohn-Nirenberg type global quantization theory in the setting of graded nilpotent Lie groups in terms of their representations. It contains a detailed exposition of related background topics on homogeneous Lie groups, nilpotent Lie groups, and the analysis of Rockland operators on graded Lie groups together with their associated Sobolev spaces. For the specific example of the Heisenberg group the theory is illustrated in detail. In addition, the book features a brief account of the corresponding quantization theory in the setting of compact Lie groups.



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Hardy Spaces on Homogeneous Groups



G. B. Folland and E. M. Stein: "That (homogeneous) groups form a natural habitat for extensions of many of the objects studied in Euclidean harmonic analysis can be understood from several points of view. First, it is a trite observation that many of the basic operators of harmonic analysis in \mathbb{R}^n are of the convolution type; and in addition these have some natural invariance with respect to dilations. If we seek to generalize this situation we are naturally led to groups which have appropriate one-parameter groups of automorphisms acting like dilations. These are the homogeneous groups."

American mathematician who was a

analysis. Picture from Wikipedia.

leading figure in the field of harmonic

Rockland operator

mathematics at the University of

Washington. Picture from Wikipedia

Rockland condition: Let T be a left-invariant differential operator on a Lie group \mathbb{G} . Then T satisfies the Rockland condition when

• for each representation $\pi \in \mathbb{G}$, except for the trivial representation, the operator $\pi(T)$ is injective on the space of smooth vectors $\mathcal{H}_{\pi}^{\infty}$, that is,

$$\forall v \in \mathcal{H}_{\pi}^{\infty} \ \pi(T)v = 0 \Rightarrow v = 0,$$

where $\pi(T)$ is the infinitesimal representation of T as an element of the universal enveloping algebra.

Rockland operator: Let \mathbb{G} be a homogeneous Lie group. A *Rockland operator* \mathcal{R} on \mathbb{G} is a left-invariant differential operator which is homogeneous of positive degree and satisfies the Rockland condition.

Helffer-Nourrigat: A left invariant homogeneous differential operator is a Rockland operator if and only if it is hypoelliptic.

Ferran Sunyer i Balaguer Prize

This monograph is the winner of the 2014 Ferran Sunyer i Balaguer Prize, a prestigious award for books of expository nature presenting the latest developments in an active area of research in mathematics, where the authors made an important contribution.

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Progress in Mathematics 314

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Quantization on Nilpotent Lie Groups



Ferran Sunyer i Balaguer Award winning monograph

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