

Description

The aim of Spectral Geometry of Partial Differential Operators is to provide a basic and self-contained introduction to the ideas underpinning spectral geometric inequalities arising in the theory of partial differential equations. Historically, one of the first inequalities of the spectral geometry was the minimisation problem of the first eigenvalue of the Dirichlet Laplacian. Nowadays, this type of inequalities of spectral geometry have expanded to many other cases with numerous applications in physics and other sciences. The main reason why the results are useful, beyond the intrinsic interest of geometric extremum problems, is that they produce a priori bounds for spectral invariants of (partial differential) operators on arbitrary domains.

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Spectral geometry

Let us consider **Riesz potential operators**

$$(\mathcal{R}_{\alpha,\Omega}f)(x) := \int_{\Omega} |x-y|^{\alpha-d} f(y) dy, \quad f \in L^2(\Omega), \quad 0 < \alpha < d, \quad (1)$$

where $\Omega \subset \mathbb{R}^d$ is a set with finite Lebesgue measure.

Rayleigh-Faber-Krahn inequality: The ball Ω^* is the maximiser of the first eigenvalue of the operator $\mathcal{R}_{\alpha,\Omega}$ among all domains of a given volume, i.e.

$$0 < \lambda_1(\Omega) \leq \lambda_1(\Omega^*)$$

for an arbitrary domain $\Omega \subset \mathbb{R}^d$ with $|\Omega| = |\Omega^*|$.

Hong-Krahn-Szegő inequity: The maximum of the second eigenvalue $\lambda_2(\Omega)$ of $\mathcal{R}_{\alpha,\Omega}$ among all sets $\Omega \subset \mathbb{R}^d$ with a given measure is approached by the union of two identical balls with mutual distance going to infinity.

Richard Feynman's drum



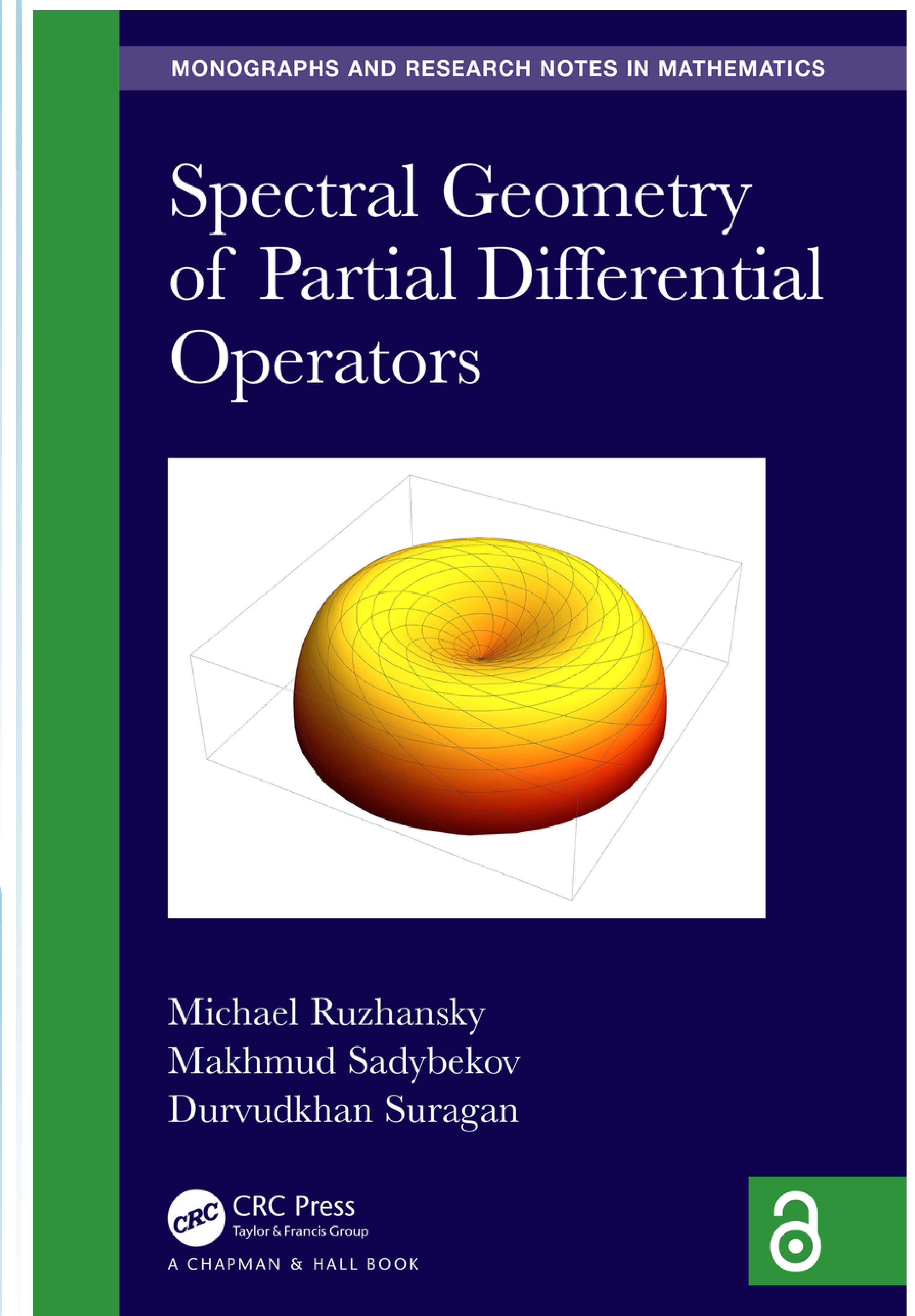
The picture from planksip.org

It is proved that the deepest bass note is produced by the circular drum among all drums of the same area (as the circular drum). Moreover, one can show that among all bodies of a given volume in the three-dimensional space with constant density, the ball has the gravitational field of the highest energy.

Rex is minimising the strain energy by circling



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Information



QR code for downloading the book
michael.ruzhansky@ugent.be
sadybekov@math.kz
durvudkhan.suragan@nu.edu.kz
Please visit:
ruzhansky.org and analysis-pde.org