UK Network on Hyperbolic Equations and Related Topics, 2020-2021

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Departments

School of Mathematics, The University of Edinburgh, Maxwell Institute for Mathematical Sciences

Department of Mathematical Sciences, Loughborough University

School of Mathematical Sciences, Queen Mary University London

Organisers

Pieter Blue (Edinburgh)
Claudia Garetto (Loughborough)
Michael Ruzhansky (Queen Mary London)

Linear and nonlinear hyperbolic partial differential equations (PDEs) arise in basically all sciences (physics, chemistry, medicine, engineering, astronomy, etc.). In physics, they model several important phenomena, from propagation of waves in a medium (for instance propagation of seismic waves during an earthquake) to refraction in crystals and gas-dynamics. The purpose of this UK network on hyperbolic equations and related topics is to bring together the expertise on hyperbolic equations of three different mathematics department (Edinburgh, Imperial, Loughborough), to strengthen the existing research collaborations and to create new ones. Three 1-day workshops per year are planned focused on different approaches to hyperbolic equations and related topics (inverse problems, kinetic theory, imaging, microlocal analysis, general relativity, etc.).

Webinar on Wednesday 25/11/2020

Join the meeting

Timetable

10-11 Dejan Gajic (Cambridge)
11-12 Michela Ottobre (Heriot Watt)

Break

13-14 Zoe Wyatt (Cambridge)
14-15 Bolys Sabitbek (Queen Mary University London)

Titles and abstracts

Dejan Gajic

Late-time asymptotics of linear waves on black holes

I will discuss a recently discovered connection between the presence of late-time tails in the dynamical behavior of black hole solutions to Einstein's equations of general relativity and the existence of conservation laws "at infinity". Understanding late-time tails is important for determining the nature of singularities inside dynamical black holes. I will focus in particular on new work, obtained in collaboration with Y. Angelopoulos and S. Aretakis, that addresses the effects of the rotation of black holes on the character of late-time tails.

Michela Ottobre

Averaging for fast non-mean-field interacting particle systems

We study a population of N particles, which evolve according to a diffusion process and interact through a dynamical network. In turn, the evolution of the network is coupled to the particles' positions. In contrast with the mean-field regime, in which each particle interacts with every other particle, i.e. with $O(N)$particles, we consider the a priori more difficult case of a sparse network; that is, each particle interacts, on average, with $O(1)$particles. We also assume that the network's dynamics is much faster than the particles' dynamics, with the time-scale of the network described by a parameter $\epsilon>0$. We combine the averaging ($\epsilon\to 0$) and the many particles ($N\to\infty$) limits and prove that the evolution of the particles' empirical density is described (after taking both limits) by a non-linear Fokker-Planck equation; we moreover give conditions under which such limits can be taken uniformly in time, hence providing a criterion under which the limiting non-linear Fokker-Planck equation is a good approximation of the original system uniformly in time. The heart of our proof consists of controlling precisely the dependence in N of the averaging estimates.

Zoe Wyatt

Coupled wave--Klein-Gordon equations in two and three spatial dimensions

Semilinear wave equations in three spatial dimensions with wave--wave nonlinearities exhibit interesting and well-studied phenomena: from John's famous blow-up example, to the null condition of Christodoulou and Klainerman, and more recently to the weak null condition of Lindblad and Rondianski. The study of coupled semilinear wave and Klein-Gordon equations is less well-developed, and interesting problems occur across the possible spectrum of wave--wave, wave--KG and KG--KG interactions. In this talk I will discuss two results, in collaboration with Shijie Dong (Fudan), where we investigate the small data global-existence of semilinear wave--Klein-Gordon equations in two and three spatial dimensions.

Bolys Sabitbek

Geometric Hardy and Hardy-Sobolev inequalities on a stratified group.

This work is devoted to present the geometric Hardy and Hardy-Sobolev inequalities for the sub-Laplacian in the half-spaces of the Carnot group and the Heisenberg group with a sharp constant. This result answers a conjecture posed by S. Larson. As a consequence, a geometric Hardy-Sobolev-Maz'ya inequality is recovered. Also, we present the geometric Hardy inequalities on the starshaped sets of the stratified group.