# Online workshop on hyperbolic equations and related topics

Thursday 09/12/2021 on zoom:

https://us02web.zoom.us/j/88645590199?pwd=NzhGZlkyTlJCSk5oZWJOWDh0TmxNdz09

Meeting ID: 886 4559 0199
Passcode: 569414

9:30 Opening

10:00-11:00 **Juan Valiente Kroon** (QMUL)

Degenerate symmetric hyperbolic systems, conformal structure and totally characteristic hypersurfaces

11:00-12:00 **Marica Minucci** (QMUL)

A conformal approach to the stability of Einstein
spaces with spatial sections of negative scalar curvature and beyond

12:00-13:00 **Giovanni Taglialatela** (Bari)

The Cauchy problem for properly hyperbolic equations in one space variable

13:00-14:00 Break

14:00-15:00 **Marianna Chatzakou** (Ghent)

Very weak solutions of hypoelliptic equations

15:00-16:00 **Priscila Leal-Da-Silva** (Loughborough)

Existence and uniqueness of solutions for the rotation-Camassa-Holm equation

**Juan Valiente Kroon** (QMUL)

Degenerate symmetric hyperbolic systems, conformal structure and totally characteristic hypersurfaces

One of the outstanding open problems in the analysis of isolated systems in General Relativity is the so-called "problem of spatial infinity". Namely, a degeneracy of the evolution equations implied by the Einstein field equations near spatial infinity. More precisely, this evolution system ceases to be symmetric hyperbolic at certain caustic sets which can be understood as the regions where spatial infinity "touches" null infinity. As such, standard theory for hyperbolic systems no longer applies and one has to rely on the fine structural properties of the Einstein field equations, in particular properties related to conformal geometry, to establish the existence and regularity of solutions. Of particular importance to this end is a totally characteristic hypersurface where the whole of the evolution equations reduce to a system of interior equations.

**Marica Minucci** (QMUL)

A conformal approach to the stability of Einstein
spaces with spatial sections of negative scalar curvature and beyond

In this talk I will show how the extended conformal Einstein field equations and a gauge based on the properties of the conformal geodesics can be used to analyse the non-linear stability of Einstein spaces with negative scalar curvature. This class of spacetimes admits a smooth conformal extension with a spacelike conformal boundary. Central to the analysis is the use of conformal Gaussian systems to obtain a hyperbolic reduction of the conformal Einstein field equations for which standard Cauchy stability results for symmetric hyperbolic systems can be employed. The use of conformal methods allows to rephrase the question of global existence of solutions to the Einstein field equations into considerations of finite existence time for the conformal evolution system. This technique is eventually extended to a different class of spacetimes.

**Giovanni Taglialatela** (Bari)

The Cauchy problem for properly hyperbolic equations in one space variable.

We consider the Cauchy problem for higher order weakly hyperbolic equations; we assume that the principal symbol depends only on one space variable and the characteristic roots $\tau_j$ verify the inequality

$$\tau_j^2(x)+\tau_k^2(x)\leq M(\tau_j(x)-\tau_k(x))^2$$

for some constant $M>0$ independent of $x$.

We prove that the Cauchy problem is well-posed in $C^\infty$ if the operator with frozen coefficients is hyperbolic in the sense of Gårding.

This is joint work with S. Spagnolo.

**Marianna Chatzakou** (Ghent)

Very weak solutions of hypoelliptic equations

I will present the works on the study of the well-posedeness of a set of hypoelliptic differential equations in the graded Lie group setting as appeared in [1], [2] and [3]. In these works we employ the notion of very weak solution and prove that, the notion of the very weak solution can be applied to our settings and converges, under some assumptions, to the classical solutions of them.

[1] M. Chatzakou, N. Tokmagambetov and M. Ruzhansky,
Fractional Klein-Gordon equation with singular mass. II: Hypoelliptic case,
*Complex Var. Elliptic Equ.,* to appear,
https://doi.org/10.1080/17476933.2021.1950146 (open access),
arXiv:2105.12862 (2021).

[2] M. Chatzakou, N. Tokmagambetov and M. Ruzhansky,
Fractional Schrödinger equations with singular potentials of higher order. II: Hypoelliptic case, *Rep. Math. Phys,* to appear,
https://doi.org/10.1080/17476933.2021.1950146 (open access),
arXiv:2106.04126 (2021).

[3] M. Chatzakou, N. Tokmagambetov and M. Ruzhansky,
The heat equation with singular potentials. II: Hypoelliptic case,
*arXiv:2110.12380* (2021).

**Priscila Leal-Da-Silva** (Loughborough)

Existence and uniqueness of solutions for the rotation-Camassa-Holm equation

Since the discovery of the Camassa-Holm equation as an integrable peakon-equation, intense research has been dedicated to its generalisations of it and to the study of their solutions. Of particular interest in this talk is the rotation-Camassa-Holm equation (rCH), a nonlocally evolutive equation that takes into consideration the Coriolis force, which is typically a manifestation of rotation when Newton's laws are applied to model physical phenomena on Earth's surface. In this talk we present conditions for the existence of traveling wave solutions and show that, given an initial data in Sobolev spaces, uniqueness of local solutions is achieved. Moreover, assuming a strong condition on the McKean quantity, the solutions can be extended globally. Extensions of these results are presented, as well as further conjectures.