Workshop on hyperbolic equations and related topics
QMUL, 25/05/2022, Room MB203

10:00-11:00 **Greg Taujanskas (Cambridge)**

Peter-Weyl theory and a null form on R x S^3

11:00-11:30 **Coffee**

11:30-12:30 **Jean-Philippe Nicolas (University of Brest)**

Peeling at an extreme black hole horizon

12:30-13:00 **Mariem Magdy Ali Mohamed (QMUL)**

Asymptotic charges for spin-1 and spin-2 fields at the critical sets of null infinity

13:00-14:00 **Lunch**

14:00-15:00 **Anna Kostianko (Imperial College London)**

Sharp upper and lower bounds of the attractor's dimension for the damped Euler-Bardina equations

15:00-15:30 **Coffee**

15:30-16:30 **Matteo Capoferri (Cardiff)**

Beyond the Hodge theorem: curl and asymmetric pseudodifferential projections

16:30-17:30 **Shrish Parmeshwar (Bath)**

Global-in-Time Solutions to the N-Body Euler-Poisson System

Dinner

Abstracts

Greg Taujanskas (Cambridge)

Peter—Weyl theory and a null form on R x S^3

In the study of nonlinear wave equations, the insight that particular kinds of nonlinearities—called null nonlinearities—lead to better regularity properties of the solution has influenced much recent progress in mathematical general relativity. Klainerman and Machedon's observation in the early 90s was that specific types of nonlinearities result in cancellations in Fourier space, which by Plancherel's Theorem may be translated to a spacetime estimate on the null form; this then in turn results in an improved local existence theorem. This observation, at least at first sight, is tied to the flat structure of R^n, and does not easily generalize to curved space. Here we will discuss ongoing work on how the representation theory of SU(2) may be employed to recover Klainerman and Machedon's argument for the wave equation on R x S^3.

Jean-Philippe Nicolas (University of Brest)

Peeling at an extreme black hole horizon

Black hole horizons are normally at finite spatial distance from the exterior region, but when they are degenerate (or extreme as they are usually referred to in this case) the spatial distance becomes infinite. One can still fall into the black hole in finite proper time but the crossing sphere is replaced by an "internal infinity". Near to the horizon of an extreme Kerr black hole, the scattering properties of test fields bear some similarities to what happens at an asymptotically flat infinity. This observation triggered a natural question concerning the peeling behaviour of test fields near such horizons. A geometrical tool known as the Couch-Torrence inversion is particularly well suited to studying this question. In this talk, I shall recall some essential notions on the peeling of fields at an asymptotically flat infinity and describe the Couch-Torrence inversion in the particular case of extreme Reissner-Nordström black holes, where it acts as a global conformal isometry of the spacetime. I will then show how to extend this inversion to more general spherically symmetric extreme horizons and describe what results can be obtained in terms of peeling. This is a joint ongoing project with Jack Borthwick (University of Besançon) and Eric Gourgoulhon (Paris Observatory).

Mariem Magdy Ali Mohamed (QMUL)

Asymptotic charges for spin-1 and spin-2 fields at the critical sets of null infinity

The asymptotic BMS charges of spin-1 and spin-2 fields are studied near spatial infinity. We evaluate the charges at the critical sets where spatial infinity meets null infinity with the aim of finding the relation between the charges at future and past null infinity. To this end, we make use of Friedrichs framework of the cylinder at spatial infinity to obtain asymptotic expansions of the Maxwell and spin-2 fields near spatial infinity which are fully determined in terms of initial data on a Cauchy hypersurface. Expanding the initial data in terms of spin-weighted spherical harmonics, it is shown that only a subset of the initial data, that satisfy certain regularity conditions, gives rise to well-defined charges at the point where future (past) infinity meets spatial infinity. Given such initial data, the charges are shown to be fully expressed in terms of the freely specifiable part of the data. Moreover, it is shown that there exists a natural correspondence between the charges defined at future and past null infinity.

Anna Kostianko (Imperial College London)

Sharp upper and lower bounds of the attractor's dimension for the damped Euler-Bardina equations

The dependence of the fractal dimension of global attractors for the damped 3D Euler–Bardina equations on the regularization parameter $\alpha > 0$ and Ekman damping coefficient $\gamma > 0$ will be discussed. We present explicit upper bounds for this dimension for the case of the whole space, periodic boundary conditions, and the case of bounded domain with Dirichlet boundary conditions. The sharpness of these estimates when $\alpha \to 0$ and $\gamma \to 0$ (which corresponds in the limit to the classical Euler equations) will be demonstrated on the 3D Kolmogorov flows on a torus.

Matteo Capoferri (Cardiff)

Beyond the Hodge theorem: curl and asymmetric pseudodifferential projections

Consider the operator curl:=∗d acting on 1-forms over a connected oriented closed Riemannian 3-manifold. Put P_±:=θ±curl, θ being the Heaviside step function. The operators P_± are completely determined by the Riemannian manifold and its orientation, and they constitute an orthonormal pair of projections which decompose the Hilbert space of real-valued coexact 1-forms into two orthogonal subspaces. We prove that the operators $P_\pm$ are pseudodifferential, write down their principal and subprincipal symbols and provide an algorithm for the explicit computation of their full symbols. We then consider the operator P_+-P_- and take its pointwise matrix trace. This gives us a scalar pseudodifferential operator A which we call the asymmetry operator. We prove that A is an operator of order -3 and define its regularised operator trace. This trace is a differential geometric invariant, a measure of the asymmetry of our Riemannian manifold under change of orientation.

The arguments are based on the use of pseudodifferential techniques developed in a recent series of papers by Vassiliev and myself. During the talk, I will briefly comment on these techniques and their applications to spectral theory and to the analysis of hyperbolic propagators.

This is joint work with Dmitri Vassiliev (UCL).

16:30-17:30 **Shrish Parmeshwar (Bath)**

Global-in-Time Solutions to the N-Body Euler-Poisson System

We investigate the N-Body compressible Euler-Poisson system, modelling multiple stars interacting with each other via Newtonian gravity. If we prescribe initial data so that each star expands indefinitely, one might expect that two of them will collide in finite time due to their expansion, and the influence of gravity. In this talk we show that there exists a large family of initial positions and velocities for the system such that each star will expand for all time, but no two will touch in finite time. To do this we use scaling mechanisms present in the compressible Euler system, and a careful analysis of how the gravitational interaction between stars affects their dynamics.